Estimation of Vibration Amplitudes of a Rotor Bearing System due to Defects in Rolling Element Bearings

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Abstract

This paper estimates the vibration amplitudes of a rotor bearing system due to the presence of localized and distributed defects in rolling bearings. The rotor bearing system has been modeled as a multiple degree of freedom system. The excitation to the system has been assumed to be caused due to the presence of defects in bearing elements and has been varied depending on the type of the defect. A single local defect as well as race waviness in the inner race of a bearing has been considered in the present study. The responses due to various types of excitations have been presented in this work. Discrete spectra have been obtained for all the types of excitations. Significant components have been predicted at the harmonics of characteristic defect frequency for both the types of defect along with sidebands at the multiples of shaft frequency. Amplitudes of all these spectral components have also been obtained. Numerical results for all these components have been calculated for NJ 204 bearing. The theoretical spectra obtained for localized defect has been compared with the experimental spectra. A comparison of the results obtained for localized and distributed defects concludes the paper.

Keywords: Bearings, Distributed defect, Local defect, Vibration amplitudes

1. Introduction

Spectral analysis of vibration signal has found wide applications in Industries for quality inspection and condition monitoring. The frequency spectra obtained from a defective bearing contain peaks at the characteristic defect frequencies of various bearing elements and can thus readily identify whether the defect is on the outer race, inner race or on a rolling element. The expressions for characteristic frequencies are well-established (Table 1). It has been found that many of these defect frequencies of localized and distributed type coincide with each other. As a result it becomes difficult to identify from frequency information whether the peak at a particular frequency is due to localized or distributed defect. Hence a study of the amplitude in addition to the frequency information of vibration response of defective bearings assumes importance.

In separate studies [1, 2], attempts were made to predict amplitudes of spectral components for vibration response from a rotor bearing system due to distributed and localized defects in various bearing elements. In the present study, a comparison of vibration amplitudes of rotor bearing system due to localized and distributed defects have been presented with specific application of inner race defect in both the cases. Numerical results for these spectral components have been calculated and a comparison of these results for different types of defects has been presented in the paper. A comparison of predicted spectra with the experimental results for localized defects has also been included in this study.

Table 1. Characteristic defect frequencies for a stationary outer race; ω_s : shaft rotation frequency; d : rolling element diameter; D : pitch diameter; Z : number of rolling elements; α : contact angle.

Characteristic frequency	Expression
Cage frequency, ω_c	$(\omega_s/2)[1-(d/D)\cos\alpha]$
Outer race defect frequency, ω_{od}	$(Z\omega_s/2)[1-(d/D)\cos\alpha]$
Inner race defect frequency, ω_{id}	$(Z\omega_s/2)[1+(d/D)\cos\alpha]$
Rolling element defect frequency, ω_{red}	$(D\omega_s/d)[1-(d^2/D^2)\cos^2\alpha]$

2. Problem formulation and solution

The rotor bearing system has been modeled as a discrete spring-mass-dashpot system following the model developed by White [3] in which the races have been assumed to be rigidly mounted on the shaft and housing and their flexural vibration has been neglected. The rotor bearing system shown in Fig. 1 has been considered for the present work and the same has been modeled as a three DOF system as shown in Fig. 2.

2.1 Elements of vibratory system

For the measurement of radial vibration at the test bearing, the shaft may be assumed to be fixed at the right support bearing (Figure 1) and the extended portion may be assumed to act as a cantilever. Stiffness, k_1, of the present model represents the stiffness of the cantilever portion of the shaft. The mass at the inner race, m_1, is the sum of the mass of the inner race and the effective mass of the extended portion of the shaft at the point of suspension of the test bearing. k_2 and c represent the stiffness and damping coefficient of the bearing. The stiffness is dependent on the load distribution among various elements and on the relationship between maximum rolling element load and the applied load. Damping coefficient, c, depends on the oil film that builds up during rotation under elasto-hydrodynamic lubrication and the extent of the load zone in the bearing. The expressions for k_2 and c have been developed in Ref. 1 and 2. The mass, m_2, is the combined effect of the mass of outer race and the mass of the housing. The loading arrangement of the rotor bearing system has considerable deflection in the bending mode. Therefore the lever has been assumed as a spring in the vibratory system under consideration. The stiffness of this spring, k_3, can be determined by assuming the lever to be a cantilever rigidly attached at one end to the test bearing housing. The mass, m_3, is the sum of the mass due to the load actually applied at the free end of the lever and the effective mass of the cantilever portion at that point.

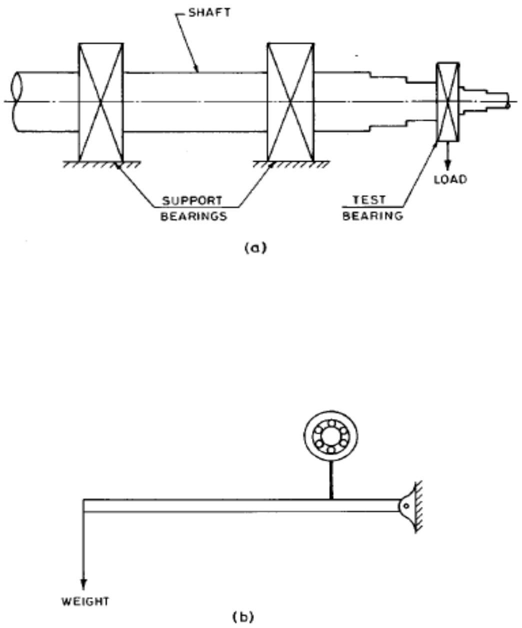

Figure 1.A schematic diagram of (a) the rotor bearing system; and (b) the loading arrangement

2.2 Equations of motion

Taking care of the different elements used in the physical model (Fig. 2), the equation of motion can be expressed as

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} c & -c & 0 \\ -c & c & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} +$$

$$\begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} F_1(t) \\ F_2(t) \\ F_3(t) \end{Bmatrix} \quad (1)$$

where F_i (i = 1,2,3) is the excitation force caused due to the defect at time t and x_i is the resultant displacement at the same time t.

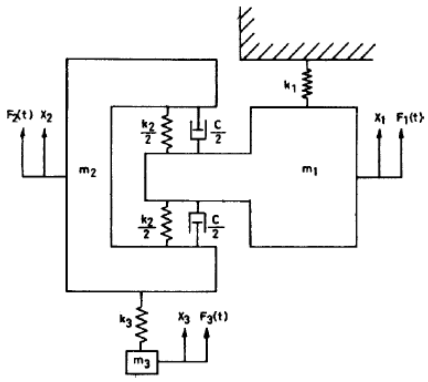

Figure 2 The three-DOF model representing the rotor bearing set up

2.3 Excitation due to local defect

The excitation to the system is caused by pulses generated due to the interaction of the defect with the mating element. When a defect on the inner race is struck by a rolling element, excitation force F_1 is resulted by generation of a pulse on the inner race or mass m_1. F_2 and F_3 are zero because a localized defect on the inner race can not cause any force to be exerted on the masses m_2 and m_3. Under this condition the pulses are generated at the inner race defect frequency, ω_{id} and the defect itself moves at the shaft speed ω_s. The load at the point of excitation and the location of the defect certainly influence the excitation force. Therefore the excitation force can be expressed as [2]:

$$F_1(t) = f(\omega_{id}\, t)\, P(\omega_s t) \cos \omega_s t \qquad (2)$$

Pulse form f and the load P have periodicity of ω_{id} and ω_s respectively. The resultant $F_1(t)$ is, therefore, a sum of harmonic components at ω_s, ω_{id} and their multiples and the sidebands about ω_{id} at multiples of shat frequency ω_s. The excitation force vector for an inner race defect is also a sum of harmonic components having the same frequencies as that of $F_1(t)$.

2.4 Excitation due to distributed defect

The distributed defect on a bearing element causes additional deflection (in addition to the static deflection due to the applied load) of a rolling element and this, in turn, results in additional spring force which is exerted on both the races. The additional spring force is time dependent and acts as the source of excitation. The excitation force applied by a rolling element at angle φ can be expressed as [1]:

$$F_{e\varphi} = K_d n \delta^{n-1}(\varphi)\, w(\varphi) \qquad (3)$$

Where K_d is deformation constant [4], n = 3/2 for ball bearing and 10/9 for roller bearing, $\delta(\varphi)$ is the deflection of a rolling element at angle φ [4] and $w(\varphi)$ is the additional deflection at the same angle φ due to race waviness. Due to waviness of order m and amplitude β at a the inner race which rotates at a speed of ω_s, the additional deflection of a rolling element at position $\omega_c t$ has a frequency of m $(\omega_c - \omega_s)$ and can be expressed as

$$w(\omega_c t) = \beta \cos m(\omega_c - \omega_s)t \qquad (4)$$

Deflection $\delta(\varphi)$ and $\delta^{n-1}(\varphi)$ are periodic and even. At $\varphi = \omega_c t$, $\delta^{n-1}(\varphi)$ can be expressed as [1]:

$$\delta^{n-1}(\varphi) = \delta_0 + \sum \delta_r \cos \omega_c t \qquad (5)$$

The forces exerted by a rolling element on the inner and outer races are equal and opposite. Therefore the total excitation forces F_1 and F_2 are also equal and opposite and are obtained by linearly adding the additional spring forces applied by all the rolling elements in the direction of radial load. F_1 and F_2, thus obtained are sum of the harmonic components and the amplitudes of these components for various frequencies are given by [1]:

$$F_1(r\omega_s) = K_d n \beta Z \frac{(\delta_{r-1} + \delta_{r+1})}{4}, \text{ for } m = r = 1, 2, 3, ... \qquad (6a)$$

$$F_1(i\omega_{id}) = K_d n \beta Z \frac{\delta_1}{2}, \qquad \text{for } m = iZ \qquad (6b)$$

$$F_1(i\omega_{id} \pm r\omega_s) = K_d n \beta Z \frac{(\delta_{r-1} + \delta_{r+1})}{4}, \text{ for } m \pm r = iZ \qquad (6c)$$

As discussed before, for any frequency ω, $F_2(\omega) = -F_1(\omega)$.

2.5 Calculation of response

Substituting the values of $F_1(\omega)$, $F_2(\omega)$ and F_3 in Eqn. 1, and applying the mechanical impedance method, the solution for any frequency ω can be found out from the following equation:

$$\{\bar{x}\} = [Z(\omega)]^{-1}\{F\} \qquad (7)$$

Where $\{\bar{x}\}$ is the phasor of the displacement vector {x} and [Z(ω)] is the impedance matrix for frequency ω and its elements may be obtained as

$$Z_{ij} = k_{ij} - \omega^2 m_{ij} + j\omega c_{ij} \qquad \text{For } i, j = 1, 2, 3 \qquad (8)$$

The amplitude of the velocity of the housing or mass m_2 at frequency ω can be obtained as the product of ω and the displacement amplitude of the housing at the same frequency.

3. Results and Discussion

In order to obtain numerical results for the velocity of the housing, an NJ 204 cylindrical roller bearing with normal clearance has been considered. The bearing has been assumed to be mounted on a shaft rotating at 1500 rpm and operating at a radial load of 750 N. For the geometry of the above bearing and spindle speed as mentioned, the important frequency components ω_s, ω_c, ω_{od} and ω_{id} are 25 Hz, 9.47 Hz, 107.14 Hz and 167.83 Hz respectively. For localized defect, a defect width of 500 micron on the inner race and for distributed defect, 30 orders of waviness having the same amplitude of 0.01 μm on the inner race have been considered. The results thus obtained have been plotted in Figure 3.

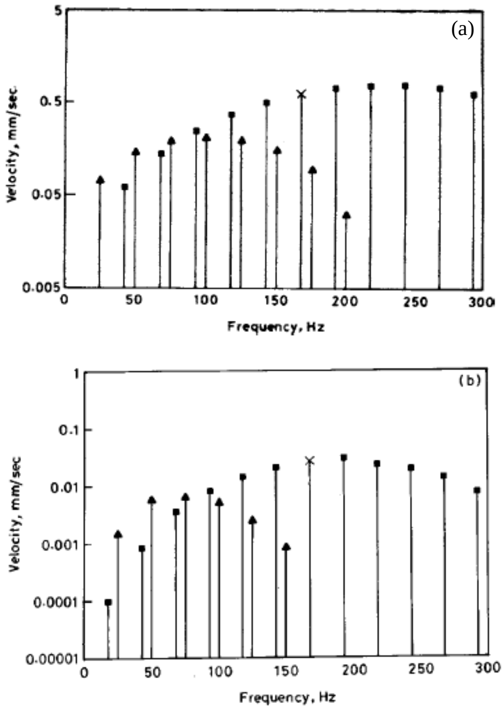

Figure 3 Frequency spectra for the velocity of the housing for (a) local and (b) distributed defects

It is observed from these figures that the spectral components for both the types of defects appear at the same frequencies. Further examination of these spectra leads to the following inferences:

- The magnitude of the response is of the same order as found in the literature for vibration response of rotor bearing system.
- Velocity response for distributed defect seems to be more evenly distributed about the characteristic frequency in comparison that for localized defect.

- The pattern of spectra for distributed defects from actual experimental measurements will be different because the amplitudes for various orders of waviness are likely to be different.

Vibration response obtained from a rotor bearing system as shown in Figure 1 with a localized defect in NJ 204 bearing has been obtained and compared with the predicted values of amplitudes of various spectral lines as discussed above. The result of the comparison is presented in Figure 4.

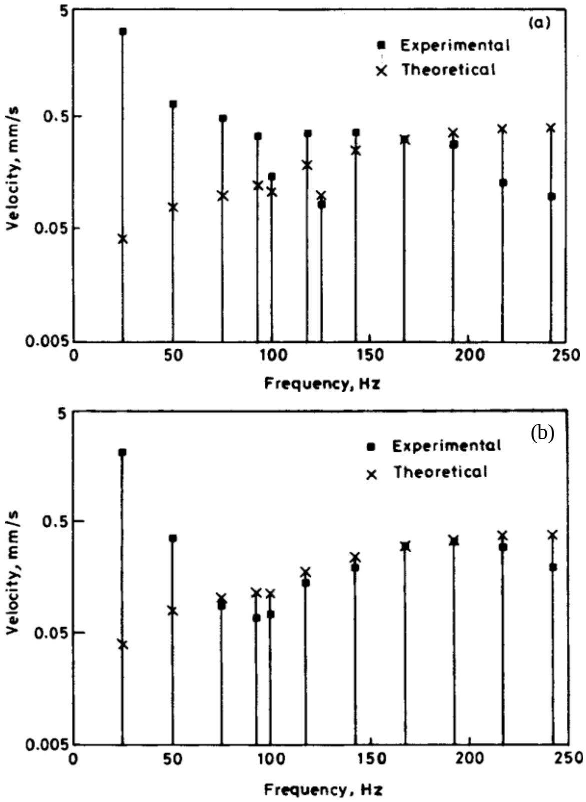

Figure 4. Comparison of theoretical and experimental values of amplitudes of spectral components for an NJ 204 bearing at 75 kg load and 1500 rpm with (a) 0.5 mm and (b) 2 mm defect on the inner race

It is observed from the above figures that there is fair agreement between amplitudes of the predicted spectra and the experimental values.

4. Conclusions

A comparative study of amplitude prediction for bearings in a rotor bearing system with localized and distributed defects on the inner race has been presented in this paper. Following conclusions may be drawn from the above study

- Spectral components for both the types of defects appear at the same frequencies justifying the need for amplitude prediction
- Theoretical values of spectral components for localized defects are in fair agreement with experimental values
- The spectrum for distributed defect is evenly distributed because same amplitude has been assumed for all orders of waviness. It is likely to be uneven for a real bearing surface.

The work can be extended in future to include multiple localized defects as well as varying amplitudes for different orders of waviness in case of distributed defects.

References

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