Determination of Creep stresses in Spherical shell

Gaurav Verma
Department of Mathematics, Gobindgarh Public College, Khanna, Punjab
email id: gk_gaurav35@yahoo.com

Abstract

The paper investigates the problem of creep stresses in the spherical shell by using Seth's transition theory. Seth's transition theory has been applied to the various problems of spherical shell under internal pressure based on creep transition. The condition of yield criterion or the associated flow rule is not assumed here. The mathematical presentation of this research paper is used commonly as design of chemical and oil plants, industrial gases and stream turbines, high speed rotating structures under effect of pressure environments. The radial and hoop stresses are calculated for the spherical shell under the creep transition. It has been observed that the value of hoop stresses is maximum at the internal surface of the spherical shell as compared to the radial stresses.

Keywords: Creep, **S**tress, Spherical shell, Pressure.

Introduction

Spherical shell structures have discovered far reaching use in present day innovation, for example, outline of synthetic and oil plants, gatherer shells, weight vessel for mechanical gases or a media transportation of high-pressurized liquids and channeling of atomic control, fast structures including streamlined warming, submerged undersea structures, earth shielded arches, and so forth. These spherical frameworks are powerful from the viewpoints of both auxiliary and structural plan In a large portion of these cases, the spherical shells need to work under serious mechanical and pressure loads, causing critical crawl and in this way lessening its administration life. The crumple or harm was started by creep, shrinkage and warm impacts or from their cooperation, on structures that both experienced or did not encounter ecological debasement. Therefore, an interest for fortifying and redesigning existing spherical structures, on account of harm caused by high pressures has been recognized[1]. Uddin [2] worked on the investigation of stability of general spherical shells under external pressure with various end-conditions. The governing non-linear differential equations for the axisymmetric deformations of spherical shells, which defines the unique states of lowest potential energy under given pressures, are solved exactly by using the method of multisegment integration. Penny [3] obtained the effects of creep in spherical shells by an analysis similar to the corresponding elastic one is described. Miller [4] evaluated solutions for stresses and displacements in a thick spherical shell subjected to internal and external pressure loads. Thakur [5] has worked on the creep transition problem of a thick isotropic spherical shell by finitesimal deformation under combined effect of temperature and internal pressure by using the concept of generalized strain measures and Seth's transition theory. Seth's transition theory does not require any assumptions like a yield condition, incompressibility condition and thus poses and solves a more general problem from which cases pertaining to the above assumptions can be worked out. This theory utilizes the concept of generalized strain measure and asymptotic solution at critical points or turning points of the differential equations defining the deformed field and has been successfully applied to a large number of problems [9-16].

Seth[6] has defined the concept of generalized strain measures as:

$$e_{ii} = \int_0^{\overset{A}{e}_{ii}} \left[1 - 2\overset{A}{e}_{ii}\right]^{\frac{n}{2}-1} d\overset{A}{e}_{ii} = \frac{1}{n}\left[1 - \left(1 - 2\overset{A}{e}_{ii}\right)^{\frac{n}{2}}\right], (i = 1,2,3) \quad (1)$$

where n is the measure and $\overset{A}{e}_{ii}$ is the Almansi finite strain components . For n = -2, -1, 0, 1, 2 it gives Cauchy, Green Hencky, Swainger and Almansi measures respectively.

Mathematical Formulation of the Problem

We consider a spherical shell, whose internal and external radii are *a* and *b* respectively, that is subjected to uniform internal pressure p_i of gradually increasing to the internal surface $r = a$ of the spherical shell. The components of displacement in spherical co-ordinate are given by Seth [6, 7]:

$$u = r(1-\beta), v = 0, \; w = 0 \tag{2}$$

where *u*, *v*, *w* (displacement components); β is position function, depending on r .

The generalized components of strain are given by Seth [7]:

$$e_{rr} = \frac{1}{n}\left[1-(r\beta'+\beta)^n\right],\; e_{\theta\theta} = \frac{1}{n}\left[1-\beta^n\right] = e_{\phi\phi}$$

$$e_{r\theta} = e_{\theta\phi} = e_{\phi r} = 0 \tag{3}$$

where *n* is measure and $\beta' = d\beta/dr$.

Stress-Strain Relation: The stress –strain relations for isotropic material are given by:

$$T_{ij} = \lambda\delta_{ij}I_1 + 2\mu e_{ij},\; (i, j = 1, 2, 3) \tag{4}$$

where T_{ij} are the stress components and e_{ij} be strain component, λ and μ are Lame's constants, $I_1 = e_{kk}$ is the first strain invariant, δ_{ij} is the Kronecker's delta.

Substituting the strain components from (equation (3)) in (Equation (4)), the stresses are obtained as:

$$T_{rr} = \frac{2\mu}{n}\left[1-(r\beta'+\beta)^n\right] + \frac{\lambda}{n}\left[3-(r\beta'+\beta)^n - 2\beta^n\right],$$

$$T_{\theta\theta} = T_{\phi\phi} = \frac{2\mu}{n}\left[1-\beta^n\right] + \frac{\lambda}{n}\left[3-(r\beta'+\beta)^n - 2\beta^n\right],$$

$$T_{r\theta} = T_{\theta\phi} = T_{\phi r} = 0 \tag{5}$$

Equation of equilibrium: The radial equilibrium of an element of spherical shell requires:

$$\frac{dT_{rr}}{dr} + \frac{2}{r}\left(T_{rr} - T_{\theta\theta}\right) = 0 \tag{6}$$

where T_{rr} and $T_{\theta\theta}$ are the radial and hoop stresses respectively.

Boundary conditions: The spherical shell under internal pressure with boundary condition:

$$T_{rr} = -p_i \text{ at } r = a,$$

$$T_{rr} = 0 \text{ at } r = b. \tag{7}$$

Critical points or Turning points: Using (Equation (5)) in (Equation (7)), we get a non- linear differential equation in β as:

$$n(2-C)P(P+1)^{n-1}\beta^{n+1}\frac{dP}{d\beta} = \left[\beta^n\left\{2-(1+P)^n\right\} - n\beta^n P\left\{(1-C)+(2-C)(1+P)^n\right\}\right] \tag{8}$$

where, $C = 2\mu/\lambda + 2\mu$ and $r\beta' = \beta P$ (*P* is the function of β and β is a function of r only). The transition points or turning point of β in (Equation (11)) are $P \to -1$ and $P \to \pm\infty$.

Analytical Solution of the Problem

For finding the creep stresses, the transition function is taken through principal stress difference (see Seth [6,7] ; Verma *et al*. [9-16]) at the transition point $P \to -1$. We define the transition function Ψ as:

$$\Psi = T_{rr} - T_{\theta\theta} = \frac{2\mu\beta^n}{n}\left[1-\left(P+1\right)^n\right] \tag{9}$$

where Ψ is a the function of r only and Ψ is the dimension.

Taking the logarithmic differentiating of (Equation (9)) with respect to r and substituting the value of $dP/d\beta$ from (Equation (8)), we get:

Taking the logarithmic differentiating of equation (9) with respect to r, we get

$$\frac{d}{dr}(\log\Psi) = \frac{nP}{r\left[1-(P+1)^n\right]}\left[1-(1+P)^n - \beta(P+1)^n\frac{dP}{d\beta}\right] \tag{10}$$

Substituting the value of $dP/d\beta$ from equation (8) and taking asymptotic value $P \rightarrow$ -1, we get:

$$\frac{d}{dr}\log\Psi = -\frac{\left[n(3-2C)+2\right]}{r(2-C)} \tag{11}$$

Integrating with respect to r, we get

$$\Psi = T_{rr} - T_{\theta\theta} = Ar^t \tag{12}$$

where $t = -\dfrac{\left[n(3-2C)+2\right]}{(2-C)}$, (13)

and A is a constant of the integration.

By using equation (12) in equation (6), we get

$$T_{rr} = -2A\int r^{t-1}dr + B \tag{14}$$

where B is a constant of integration.

Using boundary condition (i) in equation (14), we get

$$B = 2A\int_{r=b} r^{t-1}dr$$

Substituting the value of B in equation (14), we get

$$T_{rr} = 2A\int_r^b r^{t-1}dr \tag{15}$$

By using equation (15) in equation (12), we get

$$T_{\theta\theta} = 2A\left[\int_r^b r^{t-1}dr - \frac{r^t}{2}\right] \tag{16}$$

Using boundary condition (ii) in equation (19), we get

$$A = -\frac{p}{2\int_a^b r^{t-1}dr}$$

Substituting the value of A in equations (15) and (16), we get radial and hoop(circumferential) stresses as

$$T_{rr} = -p\,\frac{\int_r^b r^{t-1}dr}{2\int_a^b r^{t-1}dr} \tag{17}$$

$$T_{\theta\theta} = T_{rr} + \frac{pr^t}{2\int_a^b r^{t-1} dr}$$

It is noted that the value of $|T_{rr} - T_{\theta\theta}|$ is maximum at r = a. Therefore, the initial yielding starts at the internal surface of the spherical shell given as

$$Y = T_{\theta\theta} - T_{rr} = \frac{pa^t}{2\int_a^b r^{t-1} dr} \tag{18}$$

where Y is yield stress.

Fully plastic state:

As a particular case, we obtain the transitional creep stresses for fully plastic state of spherical shell by c approaching to zero. Therefore equations. (17) become as

$$T_{rf} = -p \frac{\int_r^b r^{t_o - 1} dr}{2\int_a^b r^{t_o-1} dr}$$

$$T_{\theta f} = T_{rf} + \frac{pr^{t_o}}{2\int_a^b r^{t_o-1} dr} \tag{19}$$

where $t_o = -\frac{3n+2}{2}$, (20)

These are expressions for radial and hoop stresses for fully plastic state. In order to calculate the Creep stresses, we introduce the non-dimensional components as following: R = r/b, R_o = a/b, $\sigma_r = T_{rr}/p$, $\sigma_\theta = T_{\theta\theta}/p$. Therefore, the equations (17) and (19) are given as

$$\sigma_{rr} = -\frac{\left(R^{-t} - 1\right)}{2\left(R_o^{-t} - 1\right)}, \sigma_{\theta\theta} = \sigma_{rr} + \frac{tR^t}{2\left(1 - R_o^{\ t}\right)} \tag{21}$$

For fully plastic state,

$$\sigma_{rf} = -\frac{\left(R^{-t_o} - 1\right)}{2\left(R_o^{-t_o} - 1\right)}, \sigma_{\theta f} = \sigma_{rf} + \frac{t_o R^{t_o}}{2\left(1 - R_o^{\ t_o}\right)} \tag{22}$$

Results and Discussion

For calculating creep stresses on the basis of above analysis, the following values have been taken, incompressible material $C = 0$, Compressible material $C = 0.25$ and compressible materials $C = 0.75$, measure n = 1/3, 1/5, 1/7 (*i . e. N* = 3, 5, 7). From equations (21) and (22), creep stresses are determined along the radii ratio R of the spherical shell for compressible as well as incompressible materials. It can be seen from figures (1-3) that the value of hoop stresses (sigma theta) are maximum at the internal surface of the spherical shell for R = 0.5 when contrasted to the radial stresses (sigma r) in the spherical shell. The value of radial stresses lie between 0 to 0.5 for all values of the measure n. This indicates that radial stresses have negligible effect on the spherical shell as compared to the hoop stresses. The maximum value of hoop stress observed is 5 for measure n = 1/3 , C = 0.75 whereas the stresses are minimum for the measure n = 1/7 , 1/5. It means that the hoop stresses are maximum for the compressible material C = 0.75 as compare to the incompressible material C = 0 for all values of the measure n = 1/7 , 1/5, 1/3.

Conclusion

Therefore, it can be concluded that the spherical shell made up of incompressible material is on secure side of design as compared to the spherical shell made up of compressible materials. It is due to occurrence of high creep stresses in spherical shell made up of compressible materials.

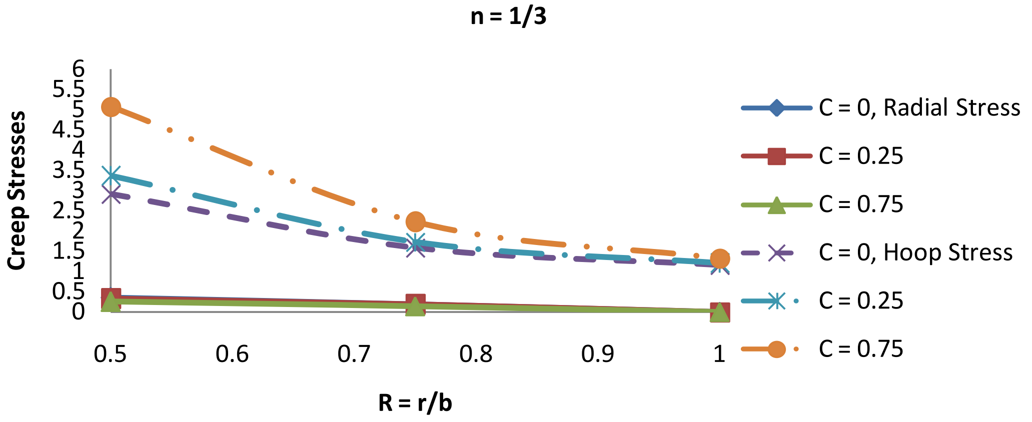

Fig. 1: Creep stress distribution in spherical shell for measure n = 1/3

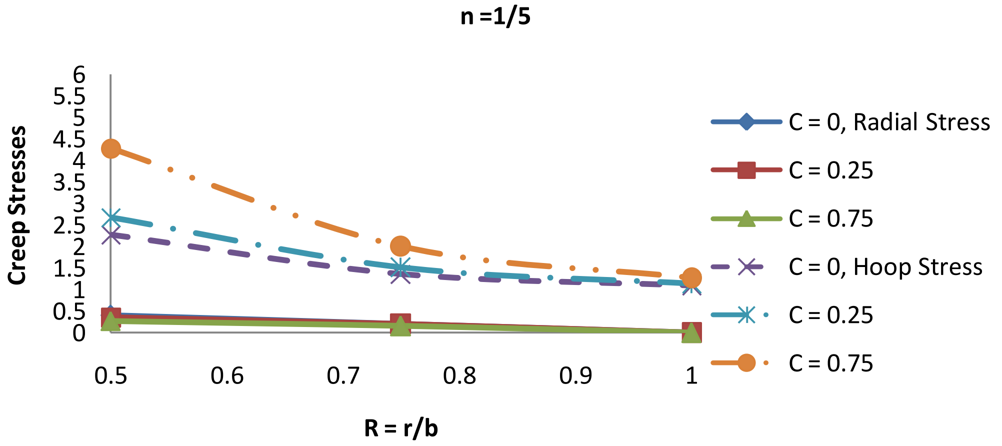

Fig. 2: Creep stress distribution in spherical shell for measure n = 1/5

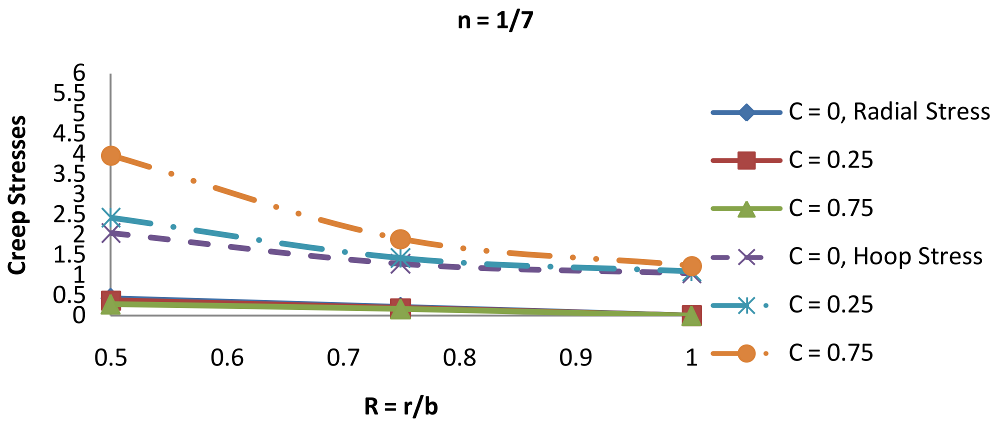

Fig. 3: Creep stress distribution in spherical shell for measure $n = 1/7$

References

1. Thakur P., Verma G., Pathania D. S., and Singh S.B., "Thermal Creep stress and strain analysis in Non-homogeneous Spherical shell", Journal of Theoretical and Applied Mechanics, Poland, 55(4), 2017, pp.1155-1165.
2. Uddin M.D., " Buckling of general spherical shell under external pressure", International Journal of Mechanical Sciences, 29(7), 1987, pp. 469-481.
3. Penny R.K., "The creep of spherical shells containing discontinuities, International Journal of Mechanical Sciences", 9(6), 1967, pp. 373-388.
4. Miller G.K., "Stresses in a spherical pressure vessel undergoing creep and dimensional changes", International Journal of Solids and Structures, 32(14), 1995, pp. 2077-2093.
5. Thakur P., "Creep Transition stresses of a Thick isotropic spherical shell by finitesimal deformation under steady state of temperature and internal pressure", Thermal Science, 15(2), 2011, S157 – S165.
6. Seth B.R., "Transition theory of elastic-plastic deformation, creep and relaxation", Nature, 195, 1962, pp. 896-897.
7. Seth B. R., "Measure concept in Mechanics", International Journal of Non-linear Mechanics, 1(1), 1966, pp. 35- 40.
8. Sokolnikoff I.S., "Mathematical theory of Elasticity", first edition, McGraw Hill book company, Inc., New York, 1946, pp. 60, 76.
9. Verma G. and Gupta S., "Elastic-plastic Transition in Shells under internal pressure", Int. J. of Emerging Technology and Advanced Engineering., 4(8), 2014, pp.126-129.
10. Verma G. and Gupta S., "Creep transition of spherical shell under internal pressure" Theoretical and Applied Science, 24, 2015, pp. 201-207.
11. Verma G., Pathania D.S. and Thakur P., 'Elastic-plastic stress analysis in a Spherical shell under internal pressure and steady state temperature", Structural Integrity and Life, Serbia, 17(1), 2017, pp. 39-43.
12. Verma G., Pathania D.S., and Thakur P., "Elastic-plastic transition on raotating spherical shells in dependence of compressibility", Kragujevac J. Sci. 39, 2017, pp. 5-16.
13. Verma G., Rana P., Pathania D.S. and Thakur P., "Creep transition in the rotating spherical shell under the effect of density variable by Seth's transition theory", AIP Conference Proceeding, 1802, 2017, pp. 020020.
14. Verma G., "Effect of external pressure on spherical shell", International Journal of Engineering Science, 24, 2017, pp. 61-73.
15. Thakur P., "Steady thermal stress and strain rates in a circular cylinder with non homogeneous compressibility subjected to thermal load", Thermal Science, 18 (1), 2014, pp. 81-92.
16. Thakur P., Singh S.B. and Kaur J., "Thermal Creep stresses and strain rates in a circular Disc with shaft having variable density", Engineering Computation, 33(3), 2016, pp. 698-712.